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COSMIC ACCELERATION WITH COSMOLOGICAL SOFT PHONONS

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The dark energy scalar field is here presented as a mean-field effect arising from the collective motion of interacting structures on an expanding lattice. This cosmological analogue to solid-state soft phonons in an unstable crystal network is shown to produce cosmic acceleration while mimicking phantom equation of state.

Keywords: Dark Energy; Cosmic Structures; Soft Phonon; Lattice Universe

1. The Accelerating Universe

Thanks to many independent observations [1–5], it is now a well-established fact that the Universe is currently undergoing a phase of accelerated expansion. The most minimal way to account for this acceleration is by using the well-known cosmological constant. Adding this constant to the model comes however with a number of problems of interpretation. One way to circumvent some of these is through the hypothesis of Dark Energy (DE) which is usually described by a scalar field [6, 7]. However, there is, to this day, no evidence that such a field, with properties suitable for producing cosmic acceleration, exists in nature at the fundamental level. It is very possible that the field might instead describe a kind of global behaviour emerging on cosmological scales whose origin is to be found in the interplay of less exotic ingredients on smaller scales. This is the view we are following in this work.

The concept of emergence is really important in condensed matter physics where we are used to manipulate quantities such as temperature and pressure that make sense only on large scales (when the number of particles involved is much bigger than, say, the Avogadro number) but whose real physical interpretation is found in microphysics. More precisely, the macroscopic notions of heat transfer or deformation involve the use of the concept of phonons. Those are described as fundamental vibrations of a lattice constituting the structure of a solid.

In this proceeding, we give a summary of the work carried out by the authors in [8]. In order to model the global dynamical properties of large scales structures, we assume a local distribution of structures regularly spread through the Universe (see Fig.1).

While this assumption might sound rough at first, it has been shown recently in [9], that such a toy model is sophisticated enough to account for many of the known cosmological properties of the Universe. The building and the numerical analysis of the model will be the object of the second section. In the third, we will be concerned with exposing the resulting features of what we call the Soft Phonon Model and its deviations from ADCM.
2. The Soft Phonon-Model

Cosmic structure formation in an expanding Universe is a complex non-linear mechanism. Instead of using Einstein’s equations in this context, we suggest a heuristic approach by constructing an effective field theory for the phonon in a FLRW Universe. Let us consider the Lagrange function describing a set of masses undergoing long-ranged interactions in such a way that each mass feels a potential generated by the whole network. Limiting ourselves to the quadratic terms in the potential, and taking the continuous limit, we get (for details, see [8]):

\[ L = \int d^3x \left\{ \frac{1}{2} v^2 \left( \frac{\partial \psi}{\partial t} \right)^2 - \left[ \nabla \psi \right]^2 + M^2 \psi^2 \right\}, \]  

(1)

with the field \( \psi \) describing the average displacement of the masses and the parameters \( v^2 \) and \( M^2 \) having respectively the dimensions of velocity squared and mass squared in suitable units. These two parameters are related to the values of the second derivatives of the potential on a single mass in the lattice. Owing to the cosmological principle, we assume that the displacement from equilibrium is homogenous throughout the Universe and we drop the gradient term in (1).

The Lagrangian (1) is written in comoving coordinates, that is, it is valid in a free-falling frame of reference with the expansion of the Universe. Cosmic expansion affects the strength of the long-range gravitational interactions between structures. To account for this, it is necessary to move to physical coordinates. As a consequence, the network parameters \( v \) and \( M \) can no longer be considered as constants. Since the comoving mean distance scales as \( l \sim a(t) l_0 \) (see Fig. 1.), by dimensional analysis we naturally obtain: \( \frac{1}{v^2} \sim \frac{1}{a^2}, \ M^2 \sim \frac{1}{a^2} \).

By adding the usual Einstein-Hilbert action to the action for the field \( \psi \) and applying the principle of least action we deduce the cosmological equations (in the conformal gauge, including matter and radiation):

\[ \frac{a^2}{a^2} = \frac{\kappa}{3} \left( \frac{1}{2} v^2 \frac{a^2}{a^2} + \frac{M^2}{2} \psi^2 \right) + H_0^2 \left( \frac{\Omega_m^0}{a} + \frac{\Omega_r^0}{a^2} \right), \]

\[ \frac{a''}{a} = \frac{\kappa}{6} M^2 \psi^2 + H_0^2 \left( \frac{\Omega_m^0}{a} \right), \]

\[ \psi'' = -a^2 v^2 M^2 \psi. \]  

(2)

We chose to integrate these equations numerically. The analysis of the model is the object of the next section.
3. Dynamical analysis

An interesting feature of the model relies in the evolution of the equation of state parameter \( w \) displayed on Fig. 2(a). It interpolates between \( w = -1/3 \) when the field is frozen and \( w = -1 \) in the far future. This asymptotic behaviour at late time corresponds to a de Sitter-like solution with an effective cosmological constant \( \Lambda_{\text{eff}} = \frac{3}{2} \frac{v^2 M^2}{\sigma^2} \). This imposes a constraint on the models parameters. It also undergoes a transitory phantom behaviour at present time, which is interesting considering recent observational research [2]. The third equation of (2) describes the time-evolution of the averaged displacement \( \psi \) of cosmic structures. When \( v^2 > 0 \), the behaviour is oscillatory. When \( v^2 < 0 \) we instead retrieve a monotonous behaviour that accounts for the accelerated expansion. This cosmological soft phonon thus describes a plastic deformation of the cosmic lattice.

![Graph](image1)

(a) Evolution of the equation of State parameter

![Graph](image2)

(b) Evolution of the acceleration parameter

Fig. 2.

Fig. 2(b) displays the evolution of the acceleration parameter. The model predicts a stronger yet smoother acceleration than \( \Lambda \)CDM. This property might have impact on the physics of large-scale structures formation.

References