July the 18th, 2017, PhysCon2017

Malbor Asllani & Timoteo Carletti

Desynchronize abnormal neuron behaviour to control epileptic seizures

www.unamur.be timoteo.carletti@unamur.be
Acknowledgements

IAP VII/19 - DYSCO

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The Brain (i)
The Brain (i)
**The Brain (i)**

**Synchronization** is a key issue to achieve the normal behaviour.

**The Virtual Brain Project**

**Human Brain Project**
In neurodegenerative diseases, such as Parkinson or epilepsy, abnormal synchronization induces undesired effects such as tremors and epileptic seizures.

Goal: to Reduce/control abnormal synchronization to avoid (lighten) such undesired effects.

- Administration of oral drugs (partially effective Parkinson’s disease but inefficient for nearly 1/3 of epileptic patients)
- Clinical methods (neurostimulation to modulate the neuronal activity to desynchronise the phase dynamics of neurons).
- Deep Brain Stimulation (DBS), microelectrodes are inserted in the basal ganglia.
- Transcranial Magnetic Stimulation (TMS), an external magnetic field interferes with the neuronal activity
Take home message

Our goal is to propose and study of a novel *minimally invasive neurostimulation* procedure principally *oriented* to *suppress* the *abnormal synchronization*.

It could thus be potentially used to reduce focal epileptic seizures or to deal with other neurological diseases.

We need:
- a model;
- a control strategy to reduce synchronisation;
- an operational implementation of such strategy.
A brain model

Neurons modelled as nonlinear oscillators (Stuart-Landau model)

\[ \dot{z}_k = \left( a_k + \omega_k - |z_k|^2 \right) z_k + Z_k, \]

where \( Z_k = \frac{K}{N} \sum_{j=1}^{N} A_{kj} z_j \)

\( \omega_k \): natural frequency

\( a_k \): bifurcation parameter

\(<0 \text{ stable eq, } >0 \text{ limit cycle}\)

\( K \): coupling parameter

A simplified brain model

Let $z_k = \rho_k e^{t\phi_k}$ and assume $\rho_k \sim \rho_j$ for all $k$ and $j$ (and $a_k = 1$) then

$$\dot{\phi}_k = \omega_k + \frac{K}{N} \sum_{j=1}^{N} A_{kj} \sin(\phi_j - \phi_k)$$

Kuramoto model
A simplified brain model

Let \( z_k = \rho_k e^{\imath \phi_k} \) and assume \( \rho_k \sim \rho_j \) for all \( k \) and \( j \) (and \( a_k = 1 \)) then

\[
\dot{\phi}_k = \omega_k + \frac{K}{N} \sum_{j=1}^{N} A_{kj} \sin(\phi_j - \phi_k)
\]

Kuramoto model

Order parameter

\[
Re^{\imath \Psi} = \frac{1}{N} \sum_{j=1}^{N} e^{\imath \phi_j}
\]

\[R(t)\]

\[K > K_c\]

\[K < K_c\]
A simplified brain model

Let \( z_k = \rho_k e^{\imath \phi_k} \) and assume \( \rho_k \sim \rho_j \) for all \( k \) and \( j \) (and \( a_k = 1 \)) then

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Kuramoto model

Order parameter

\[
\text{Re}^{\imath \Psi} = \frac{1}{N} \sum_{j=1}^{N} e^{\imath \phi_j}
\]

\( R(t) \)

\( K > K_c \)

\( K < K_c \)

Nil, partial and full phase-locking in an all-to-all network of Kuramoto oscillators. Phase-locking is governed by the coupling strength \( K \) and the distribution of intrinsic frequencies \( \omega \). Here, the intrinsic frequencies were drawn from a normal distribution (\( \text{M}=0.5\text{Hz}, \text{SD}=0.5\text{Hz} \)). The yellow disk marks the phase centroid. Its radius is a measure of coherence.
The Kuramoto model can be embedded in a (2N-dim) Hamiltonian system.

\[
H(\phi, I) = \sum_i \omega_i I_i - \frac{K}{N} \sum_{i,j} A_{ij} \sqrt{I_i I_j} (I_j - I_i) \sin(\phi_j - \phi_i) \equiv H_0(I) + V(\phi, I)
\]

On the invariant "Kuramoto" torus, the dynamics of \( H \) is the same as the Kuramoto model

\[
\mathcal{T}^K := \{(I, \phi) \in \mathbb{R}^N_+ \times \mathbb{T}^N : I_i = 1/2 \ \forall i\}
\]

Moreover, the Kuramoto oscillators are in a \textit{synchronous} state if and only if the Kuramoto torus is (transversally) \textit{unstable}.

Links between chaos and synchronization (ii)

\[ H^{\text{ctrl}}(\phi, I) = H_0(I) + V(\phi, I) + f_V(\phi, I) \]

\[ f_V(\phi, I) = \mathcal{O}(K^2) \]

Using the Hamiltonian Control theory one can modify the Hamiltonian system by adding a small term capable to increase the stability of the invariant Kuramoto torus.


Effective control

$\mathbf{f}_V(\phi, \mathbf{I})$

- Control only $M << N$ nodes and add a tunable parameter $\gamma$
- Consider a local field generated by the controlled nodes

Effective control
Result for the Kuramoto

\[ K \sim 0.1 < 0.4 \sim K_{\text{crit}} \]

original system

controlled system

[Graph showing oscillations in R for different values of K.]
Result for the Kuramoto

\[ K \sim 0.1 < 0.4 \sim K_{\text{crit}} \]

original system

controlled system

\[ K \]

\[ 0.15 \]

\[ 0.05 \]
Result for the Kuramoto

\[ K \sim 0.1 < 0.4 \sim K_{crit} \]

original system

controlled system
From the Kuramoto back to Stuart-Landau

\[ \dot{z}_k = (1 + \omega_k - |z_k|^2)z_k + Z_k + Z^{ctrl}_k \]
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Desynchronise abnormal neuron behaviour to control epileptic seizures
On the number of controllers and their strength